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5.6.5. Modelling Geomorphic Systems: Glacial

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The cryosphere contains a rich archive of the climate record, and is an important part of the global hydrological cycle. The investigation of the glacial behaviour informs our understanding of the forces that have shaped landscapes in glaciated terrains, and is key to developing understanding of past and future climate change and global climate teleconnections. Numerical modelling of glacial systems allows us to understand quantitatively the processes that drive glaciers. Three major model types exist: those that use ice extents to understand climate changes; those that investigate the forces that control ice dynamics; and those that investigate the erosional consequences of glaciation. Glacier models can be applied with a range of spatial extents, from individual cirque glaciers (e.g. Murray and Locke, 1989; Ballantyne, 2002; Coleman et al., 2009; Hughes, 2009) to continental ice sheets (e.g. Marshall and Clarke, 1999; Hubbard et al., 2005). The basis of any glacier model is a calculation of mass balance; the relationship between ice accumulation and ablation at a given point in time, under the current climate conditions. Mass balance controls glacier dynamics, which respond to processes operating on timescales of different orders of magnitude (e.g. climate change, tectonic uplift). Increasingly complex models require more variables to be specified as inputs, and so are more difficult to apply accurately; the model builder must decide which variables to exclude. However, if the input parameters are well constrained, results from complex models should be more robust. Numerical models can be mathematically 1- or 2-D, equivalent to what is more commonly described as spatially 2-D (e.g. along a line of section) or 3-D (e.g. a map view extent), which must include ice thickness. In this section, types of glacier model that can be applied to a range of different aspects of the cryosphere are discussed, alongside methodological concerns in applying different models, and important considerations in a modelling project. However, modelling studies of smaller glaciers that are confined by topography (i.e. valley glaciers) are the focus. For a starting point for models describing ice sheets, ice shelves and marine ice margin processes, glacier hydrology and isostatic adjustment, the reader is directed towards Petrenko and Whitworth (2002), Jamieson et al. (2008), Cuffey and Patterson (2010), Benn and Evans (2010) and references therein.

KEYWORDS: Numerical modelling; glacial; mass balance; energy-balance; glacier dynamics

Glacier reconstruction

Before a glacier model can be applied, a reconstruction of the glacier of interest is necessary to define boundary conditions such as topography, temperature change and precipitation amount. Model parameters, such as albedo, the values for the exponent and the constant to be used in Glen's Flow Law (Glen, 1955), or the regional lapse rate (typically a decrease of 4–8°C per 1000 m elevation gained), also need to be defined. Reconstruction can be based on both empirical field evidence of glacier form (e.g. Ballantyne, 2002; Kelly et al., 2004), and theoretical understanding of iceflow relationships (e.g. Benn and Hulton, 2010).

A simple tool for glacier reconstruction is the accumulation area ratio (AAR) which assumes that the accumulation area is ~65% of a glacier (a value derived from studies of modern glaciers; Meierding, 1982) to constrain the equilibrium line altitude (ELA) for a past glacier, by reconstruction from the position of terminal moraines. However, the AAR is not suitable for some smaller glaciers, and is increasingly being replaced by the more sophisticated area-altitude balance ratio (AABR) method, which considers glacier hypsometry to calculate ELA based on the location of the zero point in the glacier energy balance (Rea, 2009).

Types of glacier model

The applications of glacier models are hugely varied, and the type, as well as the spatial and temporal scale, of model used will depend on what is being investigated. Glacier models are commonly employed to investigate local ice extent (i.e. cirque and valley glaciers) and behaviour for a given period (Carr et al., 2010), possibly in combination with a geochronological study. Glacier models are also used to investigate the relationship of large ice volumes (i.e. ice caps and ice sheets) to regional/hemispheric climate change (Pollard, 2010) and ocean circulation patterns (e.g. Schmittner et al., 2002). Possible outcomes of glacier modelling discussed here include: a regional equilibrium line altitude (ELA) for a particular climate period; a recreation of the ice that generated a particular geomorphologic record; the conditions required for glacier steady state; and definition of the important drivers of glacial erosion and sediment production.

Glacier models can be implemented using a variety of computer architectures and software, and an important consideration should be the amount of computational time that will be required to produce the results needed. A complex model running on an office-specification machine may take an undesirably long time to produce the looked-for data, whereas some problems may be solved simply by using Excel[TM] (e.g. Brock and Arnold, 2000; Benn and Hulton, 2010). Different model types commonly applied to glacial systems are classified and described in this section. Glacier models may be classified according to the level of complexity in their calculations, as this defines the inputs needed to create a model and the output style generated.

Mass balance models

The positive degree day model (PDD; Braithwaite, 1995; Braithwaite and Raper, 2007) calculates the amount of melting that occurs during the melt season, determined by when air temperatures at the glacier surface are above a threshold temperature, usually 0–2˚C.

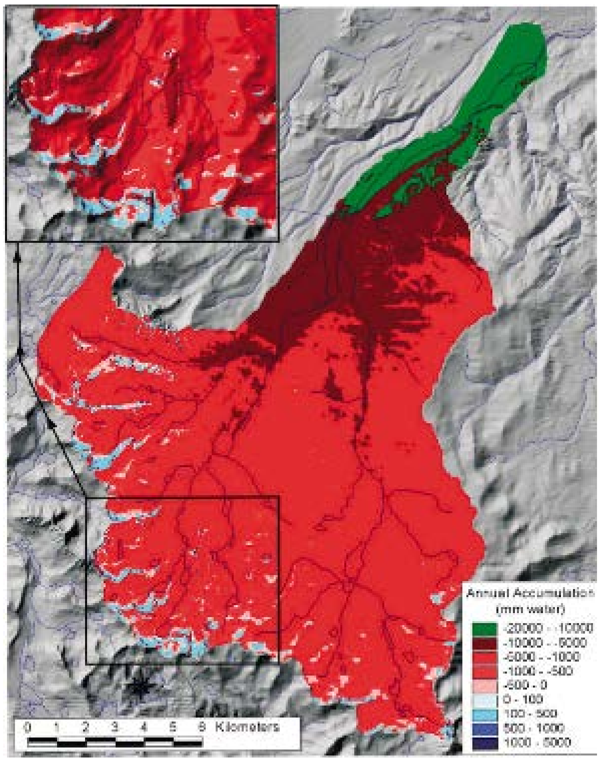

Figure 1. Result of a mass balance calculation for the Bishop Creek basin, Sierra Nevada, USA under modern climate conditions overlaid on shaded relief image of the study area. Blue indicates areas of positive net accumulation. Red indicates areas of negative net accumulation (ablation). From Plummer and Phillips (2003).

Mass balance models investigate the change in mass of a glacier and the distribution of these changes in space and time (Cuffey and Paterson, 2010) vertically relative to the glacier surface (Oerlemans, 2008, 2010). Mass balance describes when the relationship between accumulation of snow and ablation by melting and sublimation is equal to zero. A mass balance calculation can be a result in itself (Fig. 1), or used to investigate glacier behaviour as an input to an ice dynamics model. Changing the climate conditions in the mass balance model can be used to investigate the properties of palaeoglaciers.

Energy balance models calculate explicitly the energy (heat) fluxes at the glacier surface that control snow melting and sublimation that will affect mass balance (Brock and Arnold, 2000; Klok and Oerlemans, 2002; Braithwaite and Raper, 2007). The most important factor in ice surface energy flux is longwave radiation (Ohmura, 2001). Atmospheric radiative flux is strongly dependent on local topography and the annual position of the sun (Coleman et al., 2009), which varies over glacial timescales. Energy balance calculations can be used to analyse glacier sensitivity (Hubbard, 1999), which may have been very different for the same glacier throughout its history, and to investigate the effect of basin geometries on glacier behaviour. Energy balance calculations also feed into models describing ice melting that are used to understand how and when water is released from the glacial system (Hock, 2005).

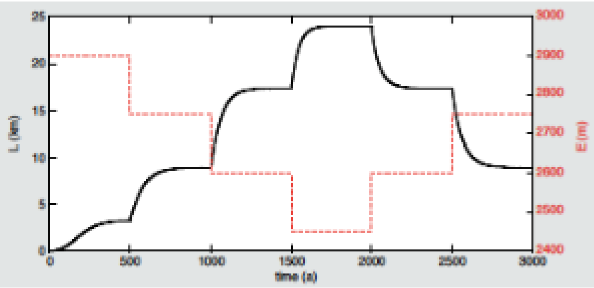

Figure 2. The response of a simple time-dependent glacier model to stepwise forcing. The steps shown by the dashed red line represent changes in ELA (E) of 150 m. Black line shows glacier length. From Oerlemans (2008).

Glacier dynamics models

The mass balance models discussed above consider glaciers in equilibrium, which is useful to understand glacier sensitivity and model consistency, but does not consider the often transient response of glaciers to external forcing (Oerlemans, 2008). Glacier dynamics models allow the investigation of time-dependent changes in glacier properties and behaviour (Oerlemans, 2008) such as change in glacier length with change in mass balance (Fig. 2). These models define the physical laws that control ice flow, and use these relationships to calculate the velocities and direction of ice flow in a glacier or ice sheet (Rutt et al., 2009). Models of ice dynamics may be very simple (e.g. Carr et al., 2010) or very complex (e.g. Huybrechts, 1990).

Glacier dynamics models are typically concerned with warm-based or polythermal glaciers or ice shelves, which flow in a much more rapid and unpredictable manner than ice bodies that are frozen to their beds. These warm-based glaciers move downstream as a result of both internal deformation and basal sliding, whereas cold-based ice bodies, such as those found inland on Antarctica, advance much more slowly, often by deformation only. The most frequently used glacier dynamics models address ice sheet and sea level response to climate change, such as the parallel ice sheet model (PISM, Fig. 3; Bueler and Brown, 2009; Winkelmann et al., 2010) and Glimmer (Rutt et al., 2009). For valley glaciers, flowline models, or 2-D iceflow models such as the Plummer and Phillips (2003) glacier model, are often used (Fig. 4).

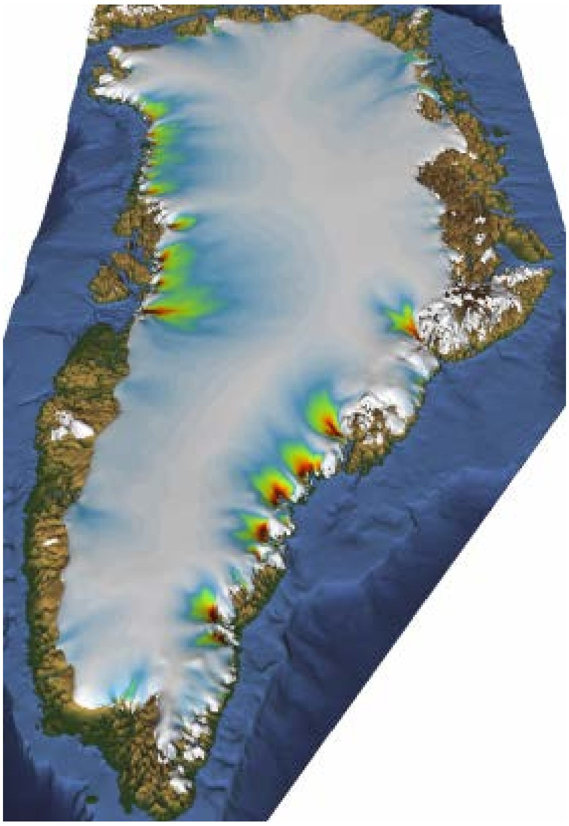

Figure 3. PISM model output for the Greenland ice sheet, showing ice surface elevation, coloured by ice thickness. From pism-docs.org.

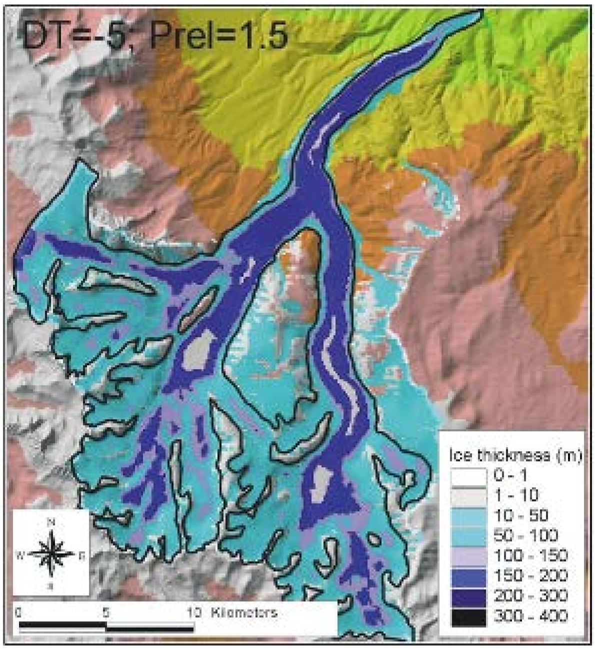

Figure 4. Iceflow model output for the Bishop Creek basin, Sierra Nevada, USA, produced by a 5°C temperature depression and a 50% increase in precipitation, equivalent to a Tioga (LGM) maximum glacier, based on field mapping of glacial trimlines and other glacial geomorphic features. From Plummer and Phillips (2003).

Flowline models (e.g. Pattyn, 1996; Oerlemans, 1997; MacGregor et al., 2000) are frequently used for valley glaciers and ice streams, to consider the transient changes that result from the forces acting along a longitudinal glacier profile (Fig. 5). Glen's Flow Law (Glen, 1955) shows that, at stresses important for normal glacier flow (50–150 kPa) the relation between shear stress change and the corresponding strain rate follows a power law (Cuffey and Patterson, 2010). Glacier dynamics models are frequently based on the shallow ice approximation (SIA; Hutter, 1983) where ice flow is driven simply by local gradients in ice surface elevation and ice thickness and all the resistance to flow is generated by the basal boundary to the ice, although development is needed to allow this model to respond well to variable bed topography (Egholm et al., 2011).

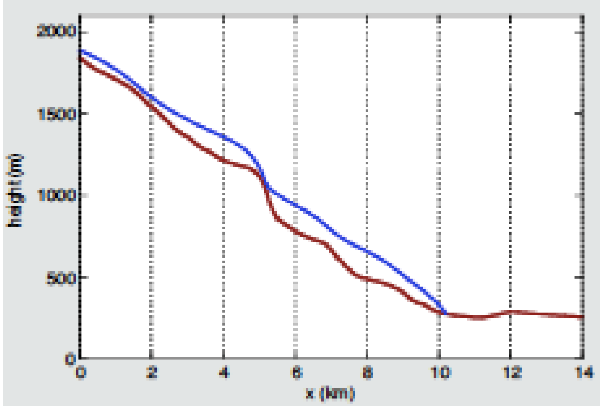

Figure 5. Flowline model of the Nigardsbreen Glacier, Norway. Blue line shows ice surface elevation relative to the glacier bed, shown by the red line, as a function of distance along a flowline (x). From Oerlemans (1997).

Glacial erosion and sediment transport models

Glacial erosion models (e.g., Braun et al., 1999; MacGregor et al., 2000; Tomkin and Roe, 2007; Tomkin, 2007, 2009; Herman and Braun, 2008; Egholm et al., 2010, 2011) investigate the complex relationships that control when and where glacial downcutting and headwall erosion occur (MacGregor et al., 2000), and the resulting landscape form (Brocklehurst and Whipple, 2007). Erosion models may represent either a valley long-profile (e.g. Brocklehurst and MacGregor, 2009), a valley cross-section (Harbor, 1992), a range cross-section (e.g. Benn and Hulton, 2010) or a swath of topography representing an entire range (Fig. 6; e.g. Egholm and Nielsen, 2010). Beneath temperate glaciers, erosion rate is usually treated as proportional to the sliding rate, but this is complicated by factors including water-pressure fluctuations and chemical dissolution (MacGregor et al., 2000). Glacier models that incorporate an erosion term usually simply define the amount of erosion as a function of the sliding rate, as this is important for quarrying and abrasion (e.g. MacGregor et al., 2000; Egholm et al., 2009) but does not take into account other erosion processes (Hooke, 1991; Alley et al., 2003).

Tectonic uplift has a crucial role to play in erosion models (Hubbard et al., 2005; Tomkin, 2007), as glaciers will respond to an increase in their elevation by becoming more erosive (Brocklehurst and Whipple, 2007;

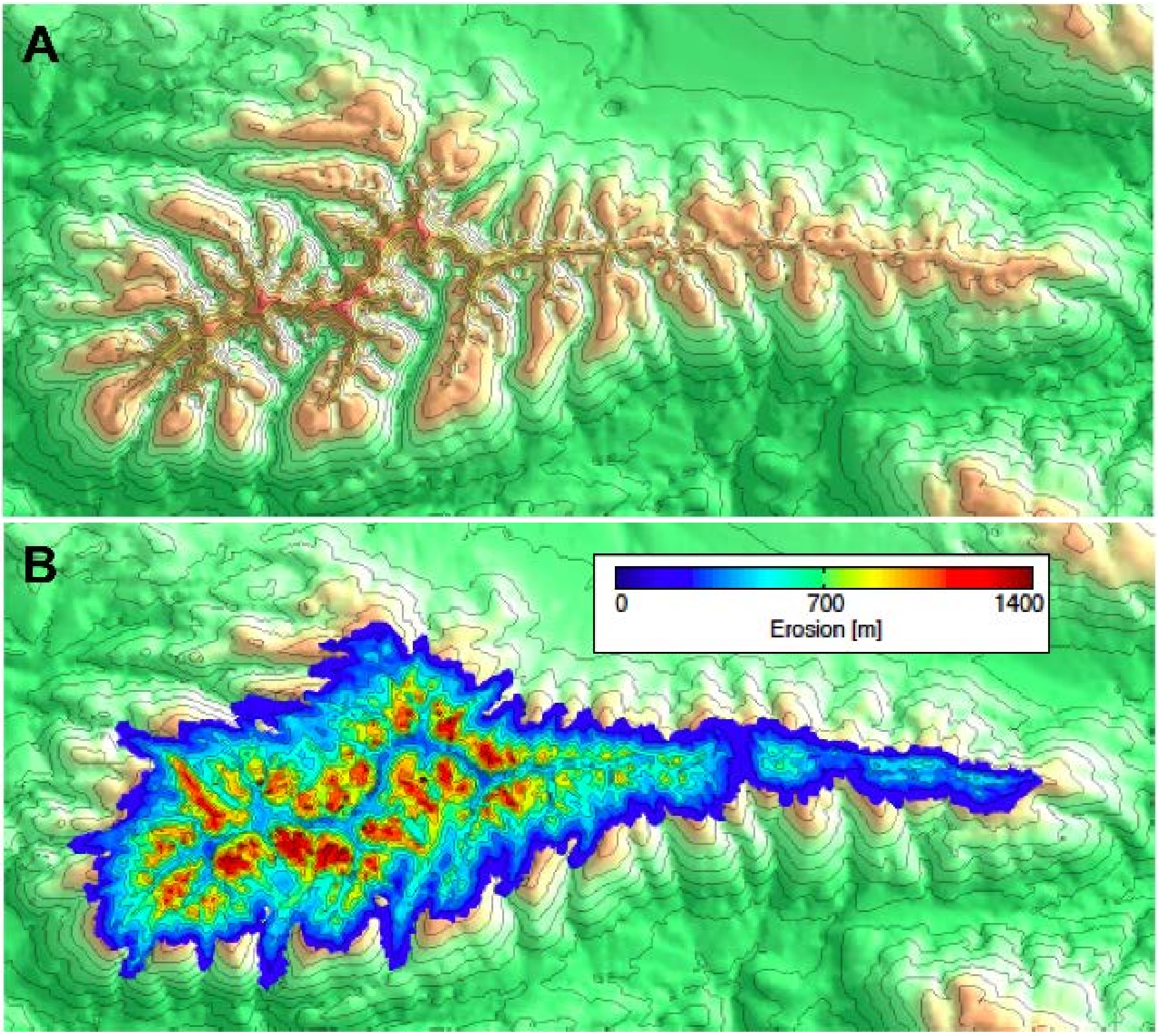

Figure 6. (A) The resulting topography after the Egholm et al. (2009) model has been applied to previously non-glaciated topography in the Sierra Nevada, Spain, for 500 ka. The postglacial DEM shows classical glacier erosion features including arêtes, cirques and hanging valleys. (B) The distribution and magnitude of erosion within the study area. Contour spacing is 200 m. From Egholm et al. (2009).

Egholm et al., 2010). Erosion is typically concentrated at the valley floor and so can lead to increased relief in glaciated landscapes, particularly when the effect of isostatic rebound is included (Molnar and England, 1990; Hubbard, 2006). However, as glacial erosion lowers the glacier bed, mass balance becomes less positive for the same climate condition and so the glacier retreats (Oerlemans, 1984; Whipple et al., 1999). These processes set up feedbacks that present challenges to glacier modelling.

Glacier sediment transport models (e.g., Dowdeswell and Siegert, 1999) consider how sediment is transported by ice and where and when it is released from the glacial system. Sediment transport is a complex problem to parameterise (Ballantyne, 2002), but results can be usefully combined with a glacier erosion model to understand the controls of glacier dynamics on sediment flux, and so interpret the resulting sedimentological record in the context of climate change. Finally, predicting the occurrence and distribution of rock glaciers remains a considerable challenge to glacier modelling (e.g Imhof, 1996; Brenning and Trombotto, 2006), in part due to the limited understanding of the processes that form rock glaciers (Haeberli et al., 2006).

Glacier model inputs

A glacier model will require a representation of the underlying topography. In the case of a model for a theoretical glacier this would require either a representative topography or flat surface as a starting point. If the modelling study considers the glaciation of a particular study area, then a representation of the real topography is required, usually as a digital elevation model (DEM). DEM resolution can be reduced from that of the raw data (typically 10-50 m cells) often without affecting the accuracy of model results (Marshall and Clarke, 1999; Evans et al., 2009). This has the advantage of decreasing model run-time. If modern ice or large volumes of sedimentary valley fill are present in the area covered by the DEM it may be desirable to create a DEM of the underlying bedrock surface (Jordan, 2009). However, this requires detailed knowledge of the thickness of the sedimentary fill, e.g. from boreholes or seismic surveying, to prevent introducing further error. It is therefore common in studies of glaciated regions to model glaciers using the DEM data as it stands. When modelling creates ice within the DEM this should be iteratively included in subsequent model runs.

DEMs typically occupy a regular grid of square cells, but models may also operate on irregular grid patterns (Fig. 7), which can increase model accuracy by removing the strong directional bias of orthogonal grid patterns, and allow decreased resolution where resolution is less important, such as unglaciated peaks (e.g. Alho and Aaltonen, 2008; Egholm and Nielsen, 2010). Cell density can then be increased at complex areas, such as glacier boundaries, to allow more detail to be captured (Egholm and Nielsen, 2010). Model grid resolution should be carefully chosen, as it can affect model output, such as the response time of glacier length to changing mass balance (Oerlemans, 2008).

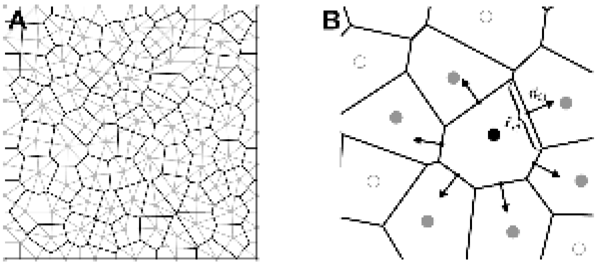

Figure 7. (A) Irregular model grid of Voronoi cells (black lines) discretised using Delaunay triangulation (grey lines) that can be used to improve model accuracy, particularly at ice body boundaries. (B) A Voronoi cell, centred on the black nodal point, Each edge of the cell has a length l_{ij} and an orientation described by the normal vector n_{ij}. From Egholm and Nielson (2010).

Temperature is one of the major factors driving changes in glacier mass balance (Anderson et al., 2010) and lapse rate varies with latitude (Syvitski et al., 2003). Rates of temperature change over the modelling period are also important, as variations in the duration and magnitude of climate change event are shown to be a primary control on glacier form (Brocklehurst and MacGregor, 2009). Precipitation may be more difficult to constrain than temperature if databases such as PRISM (Daly et al., 2002) are not available for the study area. Precipitation in alpine terrains is likely to be subject to pronounced orographic effects, causing spatially complex precipitation distributions strongly influenced by prevailing wind direction and local atmospheric conditions (e.g. Marshall and Clarke, 1999). Problems can arise in determining if precipitation falls as snow or rain (precipitation phase) and the effect of wind redistribution on solid-phase precipitation (e.g. Anders, 2008; Foster et al., 2010), which is also difficult to quantify (Gauer, 2001; Bernhardt et al., 2010).

Applications of glacier models

Some common and emerging aims when using different types of glacier model are:

- Palaeoclimate reconstructions to determine where ice occurred and the climate changes required to generate glaciers. This can be carried out using a mass and/or energy balance model, either by modelling glaciers for a particular change in climate linked to a temperature proxy such as an ice core, or by modelling glaciers to fit the observed geomorphology (Fig. 4), to define the climate change needed to match this record (Boulton and Hagdorn, 2006).

- Ice extent reconstructions to define a particular climate event (e.g. Hubbard et al., 2005), often the Last Glacial Maximum (LGM) that is considered to be the last extreme climate change event.
- Calculation of glacier sensitivity; the change in mass balance with a certain climate change (e.g. Anderson and Mackintosh, 2006).
- Erosion models can be used to consider where and when glacial erosion occurs as climate changes (e.g. Egholm and Nielsen, 2010), and so understand the landscape response to climate change events such as the Middle Pleistocene Transition (e.g. Brocklehurst and MacGregor, 2009).
- To investigate glacier hydrology and generate estimates of discharge and sediment flux from the modelled glaciers, which can be used to drive downstream sediment transport models and link these results to climate change (e.g. Richards et al., 1996).
- To remotely investigate the geomorphology of other planets, such as Mars (e.g. Pelletier et al., 2010).

Models can be designed either as discrete or finite calculations depending on their purpose. Discrete element models make calculations based on the characteristics of individual points, so in the case of a mass balance model, the mass balance is calculated for each cell of the model domain, based on the properties (e.g. elevation, degree of shading, precipitation input) of that cell. Finite element and finite difference models calculate the relationships between cells of the model grid, so in the case of an iceflow model, these calculations would consider the velocity and direction of iceflow into and out of each cell from/to neighbouring cells, allowing detailed investigation of ice dynamics (Gudmundsson, 1999). Finite volume calculations offer an alternative to some of the problems commonly encountered in finite element calculations, but may be more difficult to implement (Egholm and Nielsen, 2010).

Limitations of glacier modelling

The major limitations of many existing glacial models is that the majority of these rely on estimates of palaeoglacier properties based on assumptions about the characteristics of modern glaciers (Plummer and Phillips, 2003), and models must simplify the physical processes operating on an ice mass to describe these processes numerically (Benn and Evans, 2010). The use of assumptions about glacier properties is problematic in areas where no modern ice exists and modern ELA must also be estimated, as this introduces uncertainties into the results that are difficult to quantify. However, some models take a different approach, and reconstruct glaciers without assuming any prior knowledge of glacier form, using only local topographic and climate information as inputs (e.g. Plummer and Phillips, 2003). Uncertainty in glacier modelling exists due to the physics of ice flow being controlled by a complex set of relationships, and the wide range of processes that occur to different degrees in different glaciers. Inclusion of all these processes in a glacier model is limited by the amount of computing time that would be needed to solve these problems. For example, the SIA is not a particularly good representation of the processes that control valley glaciers (Hubbard, 2000; Egholm and Nielsen, 2010) or for ice sheets where bed topography may be pronounced (e.g. at divides and grounding zones), as it does not effectively capture longitudinal and transverse stress gradients (Pelletier et al., 2010; Egholm et al., 2011) or the contribution of basal slip to glacier movement (Gudmundsson, 2003). Models that include these stresses are called ‘higher order models’ (Pattyn et al., 2008). A further limitation of the SIA is the dependence of glacier response time on the grid resolution used (Oerlemans, 2008).

The non-linear variability of natural systems can be difficult to capture in a model. For example, the PDD model takes a typical approach to defining variability, calculating monthly mean temperature using the assumption that these values have a sinusoidal distribution around a mean value (Braithwaite, 1995). However, in reality, daily temperature patterns may vary in a more unpredictable fashion. In defining the period over which a model is to run, it is important to consider the degree of transience in ice response over the history of the glacier or ice sheet; the LGM will not be representative of an ice sheet over an entire glacial cycle (Boulton and Hagdorn, 2006). Many complicated feedbacks occur within the cryosphere and these are often difficult to constrain in a model, such as the change in albedo of snow at the glacier surface as it degrades to ice, and the effect of this on the glacier energy balance, or how increased relief due to glacial erosion will affect orographic precipitation (MacGregor et al., 2009). Incorporating these feedbacks into glacial models presents a significant challenge.

The time period over which the model runs is important; glacial-scale climate changes occur over periods of 10^3–10^6 years, but the response time of alpine glaciers may be more rapid, with steady state reached in 10^2 years or less. Temporal resolution can introduce uncertainty: depending on the chosen model timestep, climatic conditions may be averaged over a day, a month, a year or more. This leads to problems in defining the mean values for variations in temperature and precipitation over the modelling period, and is likely to remove stochastic events and seasonal variability from the model inputs. Alternatively, models may consider only stochastic events, such as surging behaviour (e.g. Bindschadler, 1982), or jokulhlaups, and their effect on glacier dynamics and hydrography (e.g. Alho and Aaltonen, 2008). The difference in timescale over which important glacier processes occur varies by orders of magnitude. Stochastic events, such as avalanches, are important controls on glacier behaviour. Avalanches will redistribute snow mass rapidly, and expose rock surfaces to erosion, but their occurrence is difficult to predict and dependent on the characteristics of the snow and the weather conditions. The dynamics of avalanching are poorly understood as a large number of controls exist (Ballantyne, 2007), so while the angle of repose of a slope can be used as a simple threshold for avalanche initiation this is not a totally satisfactory solution.

Glacier steady state may represent the end point of a model output, and occurs where accumulation is exactly equal to ablation over a period of many years and the glacier does not change in size. However, steady state rarely occurs in real glaciers (Cuffey and Paterson, 2010) so if a model depends on the assumption of steady state it may not be representative of the real glacier. Finally, verification of model results is frequently carried out by comparing the modelled glaciers to the geomorphologic record, often in the form of fragmentary terminal moraines, which may have more than one interpretation, i.e. a landslide- rather than climate- driven formation mechanism (e.g. Torvar et al., 2008; Vacco et al., 2010). However, it is possible to systematically assess the correlation between the model outputs and observed landforms automatically to improve the accuracy of these results and evaluate model performance (Napieralski et al., 2007).

Advantages and disadvantages of glacier modelling

A numerical model can be a powerful tool to reconstruct events for which there is no geological record and which would be difficult to understand quantitatively without modelling. However, uncertainties exist in any numerical model that attempts to describe the physical complexity of the natural environment. Glacier models require complex physical relationships to be defined, often using a single function. Increasing model complexity can increase calculation uncertainty, and it is vital that an estimate of uncertainty is made when modelling is undertaken, to allow understanding of the degree of confidence expressed by the model results. Moreover, it is often necessary to use output from other models to define the parameters needed (e.g. to quantify the degree of cooling at the LGM compared to the present day) and this can introduce a further level of uncertainty into calculations that must also be accounted for. Despite this, glacier models are useful to inform geomorphological studies of glaciated or deglaciated landscapes, frequently in tandem with field studies, where each aspect of the study (modelling and fieldwork) supports and validates each other, to provide a robust, quantative understanding of cryosphere behaviour, that would not be possible through fieldwork alone.

Alternative methods

Numerical models of glacial systems are commonly used for a variety of geomorphic applications, as there are few suitable alternatives. Construction of a scaled physical model of a glacier in the way that is possible in fluvial flume studies would be difficult, although some aspects of glacier behaviour can be recreated in the laboratory, such as controls on ice melting (e.g. Reznichenko et al., 2010), ice flow (e.g. Glen, 1955) and erosion and sediment transport processes at the glacier bed (e.g. Iverson, 1990, 2000). However, many studies monitor and experiment with real glaciers to investigate processes in a similar way.

The best solution where modelling is not suitable is a field-based study of modern glacier dynamics and geomorphology, with all that can be inferred from this about past glacial behaviour, although ideally an integrated approach that combines field observations with modelling is used to validate results (Hubbard et al., 2005).

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